Quantum-Controlled Few-Photon State Generated by Squeezed Atoms

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General principles and experimental schemes for generating a desired few-photon state from an aggregate of squeezed atoms are presented. Quantum-statistical information of the collective atomic dipole is found to be faithfully transferred to the photon state even in a few-photon regime. The controllability of few-photon states is shown to increase with increasing the number of squeezed atoms.

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One of the main aims in quantum optics has been to manipulate quantum-statistical properties of the electromagnetic field. Since the first milestone of generating squeezed state of light was successfully achieved [1], considerable efforts have been devoted towards the production of number state whose average photon number is less than a few tens [2,3]. If the average photon number is much greater than this, the necessity of using nonclassical light virtually disappears because the coherent state having a few tens of photons already has a sufficiently low bit error rate required for optical communication and precision measurement. Photons also carry information about the phase whose quantum fluctuations limit the interferometric sensitivity [4]. In contrast to the case of photon number, methods of regulating the phase of few-photon states have yet to be explored. In this Letter we, for the first time, present general principles and experimental schemes for generating a desired few-photon state. By this method one can control not only the average and variance in photon number, but also the width and orientation of the uncertainty ellipse in phase space in any desired direction. We will show that quantum-statistical information of the collective atom dipole is rather faithfully transferred to those of emitted photons, and discuss how to exploit this property to produce a desired few-photon state. We can thus generate any desired few-photon state by preparing the atoms in some prescribed quantum state.

We first discuss a general condition for a collection of atoms to be able to generate any desired few-photon state. Consider a simple situation in which a collection of atoms are placed in a resonant cavity and interact with a single-mode photon field. It is well-known that collective properties of two-level atoms, which are placed within the photon wavelength but not too close to avoid direct interaction between the atoms, can be described in terms of the collective spin operators as $\hat{S}_x = \sum_i \hat{\sigma}_{ix}/2$, $\hat{S}_y = \sum_i \hat{\sigma}_{iy}/2$, and $\hat{S}_z = \sum_i \hat{\sigma}_{iz}/2$, where $\hat{\sigma}_{ix}$, $\hat{\sigma}_{iy}$, and $\hat{\sigma}_{iz}$ denote the Pauli spin operators for the ith atom. Assuming the Jaynes-Cummings interaction [5] between the atoms and the photon field, the total Hamiltonian is given by

$$\hat{H} = \hbar\omega_a \hat{S}_z + \hbar\omega_f \hat{a}^\dagger \hat{a} + \hbar g(\hat{a}\hat{S}_+ + \hat{a}^\dagger \hat{S}_-), \tag{1}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators of the photon field, $\hat{S}_\pm \equiv \hat{S}_x \pm i\hat{S}_y$, $\hbar\omega_a$ is the energy difference between the two levels of the atoms, $\hbar\omega_f$ is an energy quantum of the photon, and g is a coupling constant. When $\omega_f = \omega_a$, we can eliminate the noninteracting part of the Hamiltonian $H_0 = \hbar\omega_a \hat{S}_z + \hbar\omega_f \hat{a}^\dagger \hat{a}$ by working on a rotating frame $e^{i\hat{H}_0 t/\hbar}|\psi\rangle$. Since we want to manipulate the width and the orientation of the uncertainty ellipse in phase space in any desired direction, it is convenient to introduce operators in the direction specified by the azimuth angle ϕ as $\hat{a}_\phi \equiv \frac{1}{2}(\hat{a}e^{-i\phi} + \hat{a}^\dagger e^{i\phi})$ and $\hat{S}_\phi \equiv \frac{1}{2}(\hat{S}_+ e^{-i\phi} + \hat{S}_- e^{i\phi})$. These operators obey the following equations of motion:

$$\frac{d\hat{a}_\phi}{dt} = -g\hat{S}_{-\phi+\pi/2}, \tag{2}$$

$$\frac{d\hat{S}_{-\phi+\pi/2}}{dt} = -2g\hat{a}_\phi \hat{S}_z, \tag{3}$$

$$\frac{d\hat{S}_z}{dt} = 2g(\hat{a}_\phi \hat{S}_{-\phi+\pi/2} + \hat{a}_{\phi+\pi/2}\hat{S}_{-\phi}). \tag{4}$$

When the atoms are irradiated by coherent light with classical intensity, the mean field approximation is valid, and Eqs. (2)-(4) reduce to the familiar optical Bloch equations. Since we are interested in reducing quantum fluctuations in a few-photon regime, we have to take into account higher-order correlations. In particular, we are interested in the variance of $\hat{a}_\phi$. Its dynamical evolution is governed by

$$\frac{d\langle(\Delta\hat{a}_\phi)^2\rangle}{dt} = -2g\langle(\Delta\hat{a}_\phi)(\Delta\hat{S}_{-\phi+\pi/2})\rangle, \tag{5}$$

$$\frac{d^2\langle(\Delta\hat{a}_\phi)^2\rangle}{dt^2} = g^2\Big(4\langle(\Delta\hat{a}_\phi)(\Delta\hat{a}_\phi\hat{S}_z)\rangle + 2\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle\Big), \tag{6}$$

where $\Delta\hat{\mathcal{O}} \equiv \hat{\mathcal{O}} - \langle\hat{\mathcal{O}}\rangle$ for an arbitrary operator $\hat{\mathcal{O}}$. When the field is initially in the vacuum state, there is no initial correlation between the atoms and the field, so the right-hand side of Eq. (5) vanishes at $t = 0$ and

the first term in the square brackets in Eq. (6) becomes $4\langle(\Delta\hat{a}_\phi)^2\rangle\langle\hat{S}_z\rangle = \langle\hat{S}_z\rangle$. The second derivative in Eq. (6) is therefore negative if and only if

$$\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle < \frac{|\langle\hat{S}_z\rangle|}{2} \quad \text{and} \quad \langle\hat{S}_z\rangle < 0. \tag{7}$$

In this case, the fluctuations in $\hat{a}_\phi$ will be suppressed to below the standard quantum limit at times much shorter than $\sim g^{-1}$. The field displacement $\langle\hat{a}_\phi\rangle$ and its variance $\langle(\Delta\hat{a}_\phi)^2\rangle$ can be controlled independently because their time evolutions are governed respectively by $\langle\hat{S}_{-\phi+\pi/2}\rangle$ and $\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle$. From Eqs. (2)-(6) we find that the amplitude squeezed state is obtained from the atomic state that satisfies, e.g., $\langle\hat{S}_x\rangle = 0, \langle\hat{S}_y\rangle \neq 0, \langle\hat{S}_z\rangle < 0$, and $\langle(\Delta\hat{S}_y)^2\rangle < |\langle\hat{S}_z\rangle|/2$. The phase squeezed state is obtained only by replacing $\langle(\Delta\hat{S}_y)^2\rangle$ for $\langle(\Delta\hat{S}_x)^2\rangle$. The right figures in Fig. 1 (a) and (b) illustrate generation of the amplitude squeezed state and the phase squeezed state from squeezed fifty atoms. The left figures show the initial squeezed atom states, prepared by the scheme discussed below, in the spin quasi-probability distribution defined by $\langle\theta,\phi|\hat{\rho}_{\rm atom}|\theta,\phi\rangle$, where $|\theta,\phi\rangle \equiv e^{-i\phi\hat{S}_z}e^{-i\theta\hat{S}_y}|S, S_z = S\rangle$ is the coherent state of spin or angular momentum and will be referred to as the Bloch state [6]. The quasi-probability distribution of the photon field $Q(\alpha) \equiv {\rm Tr}_{\rm atom}[\langle\alpha|\hat{\rho}|\alpha\rangle]/\pi$ (right figures in Fig. 1 (a) and (b)) is obtained by numerical diagonalization of the Jaynes-Cummings Hamiltonian, where $|\alpha\rangle$ is the coherent state of the radiation field with amplitude α, and $\hat{\rho}$ is the density operator of the entire system when the maximal squeezing is obtained. From Fig. 1 we find that the profile of $Q(\alpha)$ follows that of $\langle\theta,\phi|\hat{\rho}_{\rm atom}|\theta,\phi\rangle$ projected on the S_x-S_y plane, where S_x and S_y correspond to $-{\rm Im}\ \alpha$ and $-{\rm Re}\ \alpha$. This rather faithful transfer of quantum information from the atomic system to the photon system holds in general, and tells us how to prepare the collective atomic state in order to produce a desired few-photon state. Figure 1(c) shows the time evolutions of the atomic and field observables for the case of Fig. 1(b). The radiation-field amplitude $\langle\hat{a}\rangle$ grows as the mean spin vector tilts towards the negative z-axis (i.e., $\theta \to 0$). We also note that quantum fluctuations in the radiation field, $\langle(\Delta\hat{a}_2)^2\rangle$ decreases in time at the expense of increasing atomic fluctuations $\langle(\Delta\hat{S}_x)^2\rangle$.

For a collection of atoms to be able to radiate a photon state that is squeezed in any desired direction in phase space, which we will refer to as *tailor-made radiation*, condition (7) has to be met for arbitrary ϕ. Thus the necessary and sufficient condition for the tailor-made radiation is given by

$$\langle(\Delta\hat{S}_\perp^{\rm min})^2\rangle < \frac{|\langle\hat{\boldsymbol{S}}\rangle|}{2}, \tag{8}$$

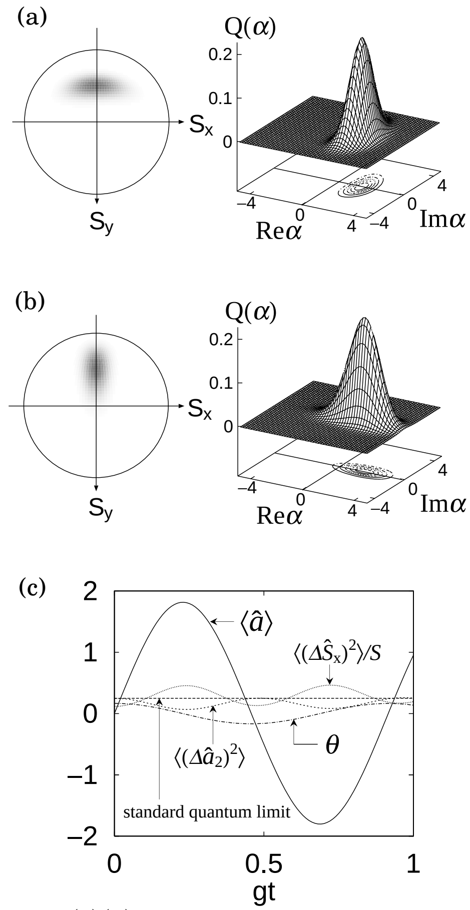

FIG. 1. (a)(b): Quasi-probability distributions of squeezed fifty atoms with $\langle(\Delta\hat{S}_\perp)^2\rangle = 2.93$, $|\langle\hat{S}\rangle| = 24.2$, $\langle\hat{S}_x\rangle = 0$, and $\theta = \pi/6$ as seen from the negative z axis (left) and those of the radiation field emitted from the atoms (right). In (a), the amplitude is squeezed, while in (b) the phase is squeezed. (c): Time evolutions of amplitude $\langle\hat{a}\rangle$ and variance $\langle(\Delta\hat{a}_2)^2\rangle$ of the radiation field and the normalized variance of the atomic dipoles for the case of Fig. 1(b). The standard quantum limit shows the value of $\langle(\Delta\hat{a}_2)^2\rangle$ for the coherent state and θ denotes the angle between the mean spin vector and the negative z axis.

where $\langle(\Delta\hat{S}_\perp^{\rm min})^2\rangle$ denotes the minimum value of the variance perpendicular to the mean spin vector $\langle\hat{\boldsymbol{S}}\rangle$. Note that condition (8) is more stringent than the condition used in Ref. [4,7] which discussed the interferometric phase sensitivity. A crucial observation is that phase squeezing (as in Fig. 1 (b)) can only be obtained by states satisfying the condition (8). This is because fluctuations projected on the S_x-S_y plane cannot be reduced to below $\langle(\Delta\hat{S}_\perp^{\rm min})^2\rangle$ in the direction of ϕ by any rotation of the spin vector. This is why the Bloch state which has an isotropic uncertainty distribution with respect to the plane perpendicular to the mean spin vector cannot radiate the phase squeezed state. The Bloch state can,

on the other hand, radiate amplitude-squeezed state by tilting the spin vector [8]. Nor can a single atom be used for the tailor-made radiation, because it has no partner to be quantum-mechanically entangled with in order to meet condition (8). It should also be noted that a popular definition of the spin squeezing [8–11]

$$\langle(\Delta\hat{S}_i)^2\rangle < \frac{|\langle\hat{S}_z\rangle|}{2} \quad (i = x \text{ or } y), \tag{9}$$

cannot be used as a criterion for the tailor-made radiation because this condition can be met by the Bloch state whose spin vector is tilted from the z axis [12].

In order to control the degree of squeezing of photons, we must solve the time evolution (2)-(6). Although the exact solutions is unavailable because of high nonlinearity of these equations, we can obtain approximate solution that becomes exact when the number of atoms is large and $\langle\hat{S}_z\rangle \sim -S$, i.e., the spin angle from the negative z-axis is small:

$$\langle(\Delta\hat{a}_\phi)^2\rangle = \frac{1}{4}\cos^2\sqrt{2S_0}gt + \frac{\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle_0}{2S_0}\sin^2\sqrt{2S_0}gt \tag{10}$$

$$\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle = \langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle_0\cos^2\sqrt{2S_0}gt + \frac{S_0}{2}\sin^2\sqrt{2S_0}gt \tag{11}$$

where $\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle_0$ denotes the variance in the initial spin state, and the length of the mean spin vector $|\langle\hat{\boldsymbol{S}}\rangle|$ is assumed to be almost constant S_0. These solutions are periodic functions with period $\pi(g\sqrt{2S_0})^{-1}$. The photon fluctuation (10) attains a minimum value $\langle(\Delta\hat{S}_{-\phi+\pi/2})^2\rangle_0/(2S_0)$ at $t = \pi(2g\sqrt{2S_0})^{-1}$, and therefore the squeezed radiation can be obtained if the spin satisfies the condition (7) and the degree of squeezing is proportional to that of the spin. The expressions (10) and (11) imply that quantum fluctuation is transferred from the field to the atoms, as can be seen in Fig. 1(c).

Several methods have been proposed to generate squeezed spin state: the Jaynes-Cummings interaction with the coherent state [13], or with the squeezed vacuum [7], and interaction of the spins through nonlinear Hamiltonians [11,4]. We focus our discussion to the first method. In Ref. [13] the Jaynes-Cummings Hamiltonian (1) with the initial coherent state and the initial spin state $|S, S\rangle$ are used, while in Ref. [7] the interaction Hamiltonian $\hat{H}_2 = \hbar\Omega(\hat{a}^\dagger\hat{S}_+ + \hat{a}\hat{S}_-)$ with the initial spin state $|S, -S\rangle$ are used, where Ω is a coupling constant. Since these models are mathematically equivalent [14], we restrict our attention to the former.

The quantity of our interest is the degree of squeezing of the spin obtained by the interaction with the coherent state, given the number of atoms. We seek the maximally squeezed spin state numerically, rotate it in various direction, and use it as the initial spin state in the radiation process. Figure 2 shows the range of amplitude $|\langle\hat{a}\rangle|$ and variance $\langle(\Delta\hat{a}_\phi)^2\rangle$ of the radiation field that can be achieved by $2 \sim 100$ atoms. This shows that the larger the number of atoms, the tunable range for the radiation field becomes wider.

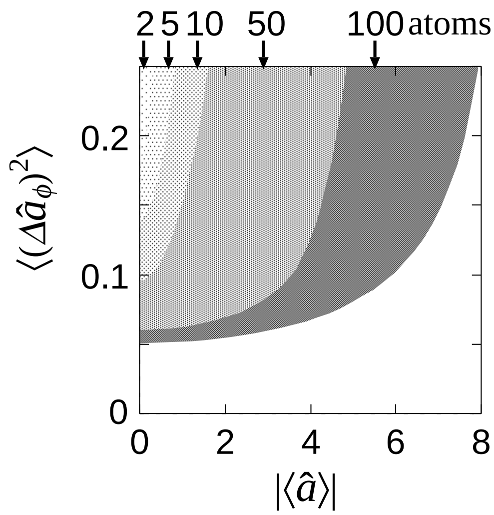

FIG. 2. Possible range of amplitude $|\langle\hat{a}\rangle|$ and its variance $\langle(\Delta\hat{a}_\phi)^2\rangle$ of the radiation field that can be obtained by 2, 5, 10, 50, and 100 atoms prepared by interaction with the coherent state.

We propose two possible experimental schemes to implement our theory. The first one is a scheme using the micromaser technique [15] as illustrated in Fig. 3, where the state of atoms is indicated in the spin quasiprobability distribution at each stage. It consists of three stages: (1) The excited two-level atoms are injected into the first cavity in which the atoms become squeezed by interacting with the coherent state of the radiation field $|\alpha\rangle$ prepared by laser or maser [13]. (2) The output squeezed atoms pass through the coherent field with classical intensity. This field rotates the mean spin vector in the spin space to the desired direction. To control the rotation axis the coherent field and the classical field must be driven synchronously with an appropriate phase difference provided by the phase shifter. (3) The atoms go into the third vacuum cavity, radiate photons and come out of the cavity before reabsorbing the emitted photons. Left in the third cavity is the desired photon state which we can take out by switching the Q-factor of the cavity mechanically or by applying a magnetic field.

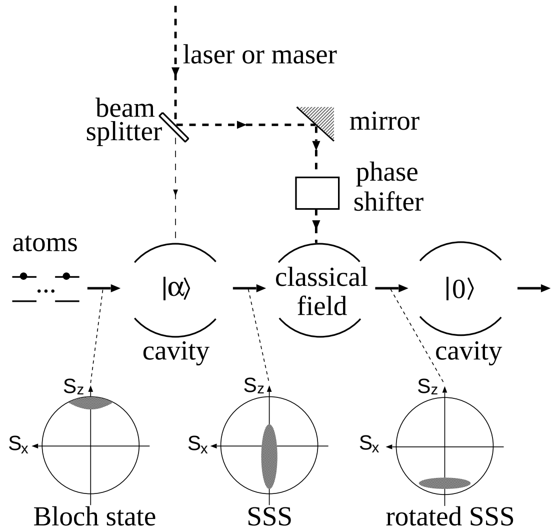

FIG. 3. Schematic illustration of an experimental setup to generate a few-photon state that features any desired quantum statistics. The state of the atoms at each stage is shown by the spin quasi-probability distribution. The two-level excited atoms go into the first cavity and interact with a coherent state of the radiation field $|\alpha\rangle$. The output atoms are in a squeezed spin state (SSS). By interaction with a classical field in the second cavity, the mean spin vector is rotated to a desired direction, where the rotation axis can be specified by the phase shifter. The atoms then go into the third cavity and emit photons there. Left in the third cavity is the desired few-photon state which can be extracted by our changing the cavity quality factor.

The second scheme is to employ the atom trapping and the laser-cooling, in which the above three stages are implemented at the same place. For this purpose, the optical cavity should be off-resonant during the preparation of the atomic state, and be resonant only at the time of radiation. Interaction with the coherent state corresponding to the first stage above can be realized by interaction with the center-of-mass oscillation of atoms through the stimulated Raman transition [16]. This second scheme has the advantage of producing a large number of squeezed atoms.

In an actual experiment, we must finish the whole sequence of processes before the two-level atoms decay into other levels. If we use, for example, the $63p_{3/2} \to 61d_{3/2}$ transition of rubidium atoms, the lifetime is of order millisecond and the coupling constant is $g \sim 10^4$ Hz. Since the required interaction time is $gt \sim 1$, i.e., $t \sim 10^{-4}$ sec, the whole procedure can be accomplished within the atomic lifetime. The finite Q-factor of the cavity will not be an obstacle, since the cavity lifetime now reaches $t_c \sim 10^{-1}$ sec in the microwave regime [2]. Thermal photons, however, must be carefully suppressed. If we use the above-mentioned transition (21.5 GHz), the temperature should be below, say, 0.2 K in order to suppress the average number of thermal photons in the cavity to below 0.01.

In conclusion, we have shown that the quantum-statistical information of collective atomic dipoles is faithfully transferred to the radiation field even in a few-photon regime. This implies that we can produce a desired few-photon state by preparing atoms in an appropriate squeezed state. This idea can be tested using a high-Q cavity that sustains more than one atom undergoing the Jaynes-Cummings interaction with the radiation field.

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is in the vacuum state, and the situation in which all atoms are excited and the field is in a coherent state. In both schemes, the atoms produce during its evolution a squeezed field with sub-Poissonian statistics, but they do not produce a phase-squeezed state. A. Heidmann, J. M. Raimond, and S. Reynaud, Phys. Rev. Lett. **54**, 326 (1985); A. Heidmann, J. M. Raimond, S. Reynaud, and N. Zagury, Opt. Commun. **54**, 54 (1985); see also, M. Butler and P. D. Drummond, Opt. Acta **33**, 1 (1986).

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[14] From the relation $e^{i\pi\hat{S}_x}\hat{S}_\pm e^{-i\pi\hat{S}_x} = \hat{S}_\mp$, we find that time evolution with $\hat{H}_2$ can be rewritten by

$$e^{-i\hat{H}_2 t}|-S\rangle = e^{-i\hat{H}_2 t}e^{-i\pi\hat{S}_x}|S\rangle = e^{-i\pi\hat{S}_x}e^{-i\hat{H}_{\rm int}t}|S\rangle,$$

which indicates that both models are equivalent rotating the initial and final spin state.

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